PHYS 301
Electricity and Magnetism

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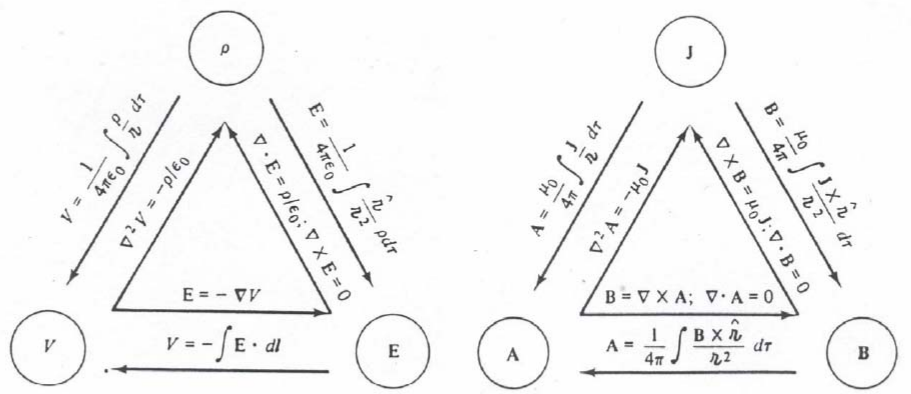
Today!

- Magnetic fields
  - Law of Biot-Savart
  - Ampere's Law

electrostatics   magnetostatics



magnetostatics

- Current …
  - *conventional current flow* = the flow of positive charges
  - A comparative name/symbol "hierarchy" between charges and currents:

electrostatics		magnetostatics	
q	*charge*	*no magnetic monopoles!*	
λ	*line charge (density)*	$I = \frac{dq}{dt} = \lvert \lambda \vec{v} \rvert = \int \vec{J} \cdot d\vec{A}$	*"current"*
σ	*surface charge (density)*	$\vec{K} = \sigma \vec{v}$	*"surface current (density)"*
ρ	*(volume) charge density*	$\vec{J} = \rho \vec{v}$	*"current density"*

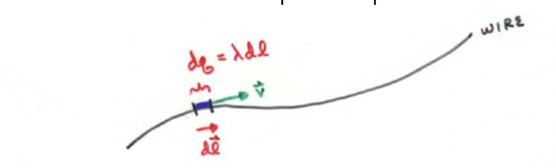
magnetostatics

THE MAGNETIC FORCE ON A CURRENT CARRYING WIRE (DUE TO AN EXTERNAL MAGNETIC FIELD) IS

$$\vec{F} = \int I\,(d\vec{l} \times \vec{B})$$

WHERE

$$I = \frac{dq}{dt} = \left|\frac{\lambda\, d\vec{\ell}}{dt}\right| = |\lambda\, \vec{v}\,|$$



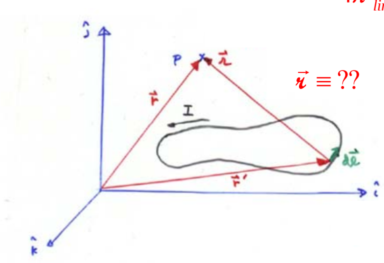
magnetostatics

THE LAW OF BIOT-SAVART

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{line}\frac{\vec{I}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,dl' = \frac{\mu_o I}{4\pi}\int_{line}\frac{d\vec{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

for constant I

$\vec{\mathscr{r}} \equiv ??$



[for steady currents!]

OR, MORE GENERALLY,

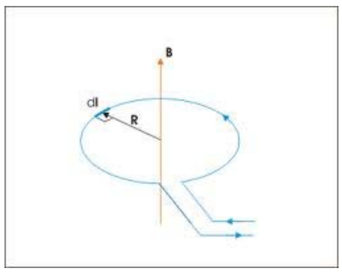
$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{vol}\frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

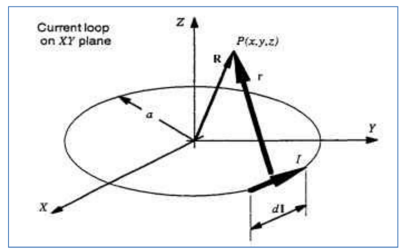
where $J = J(\vec{r}\,')$

magnetostatics

***E.g.*, the "axial" magnetic field for a circular current loop carrying a current, I, is**

$$\vec{B} = \frac{\mu_o I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\hat{k}$$





Trickier!!

magnetostatics

THE DIVERGENCE OF $\vec{B}$

Expectations: **"locally," the magnetic field for a current-carrying wire looks like this:**

? WHAT'S THE DIRECTION OF $\vec{J}$, HERE ? ⟶

? WHAT'S AN EXPRESSION FOR $\vec{J}(\vec{r})$?

[since $\vec{J} = d\vec{I}\,/\,dA_{\perp}$, then $I = \int_{surface} \vec{J}\cdot\hat{n}\,dA$]